Package 'tram'

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Title Transformation Models

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Description Formula-based user-interfaces to specific transformation models implemented in package 'mlt'. Available models include Cox models, some parametric survival models (Weibull, etc.), models for ordered categorical variables, normal and non-normal (Box-Cox type) linear models, and continuous outcome logistic regression (Lohse et al., 2017, <DOI:10.12688/f1000research.12934.1>). The underlying theory is described in Hothorn et al. (2018) <DOI:10.1111/sjos.12291>. An extension to transformation models for clustered data is provided (Barbanti and Hothorn, 2022, <arxiv:1910.09219>). Multivariate conditional transformation models (Klein et al, 2022, <DOI:10.1111/sjos.12501>) can be fitted as well.

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R topics documented:

`Aareg` *Aalen Additive Hazards Model*

Description

Aalen model with fully parameterised hazard function

Usage

```
Aareg(formula, data, subset, weights, offset, cluster, na.action = na.omit, ...)
```

Arguments

`formula`	an object of class `"formula"`: a symbolic description of the model structure to be fitted. The details of model specification are given under `tram` and in the package vignette.
`data`	an optional data frame, list or environment (or object coercible by `as.data.frame` to a data frame) containing the variables in the model. If not found in `data`, the variables are taken from `environment(formula)`.
`subset`	an optional vector specifying a subset of observations to be used in the fitting process.
`weights`	an optional vector of weights to be used in the fitting process. Should be `NULL` or a numeric vector. If present, the weighted log-likelihood is maximised.

offset	this can be used to specify an _a priori_ known component to be included in the linear predictor during fitting. This should be NULL or a numeric vector of length equal to the number of cases.
cluster	optional factor with a cluster ID employed for computing clustered covariances.
na.action	a function which indicates what should happen when the data contain NAs. The default is set to na.omit.
...	additional arguments to tram.

Details

This function allows simultaneous estimation of the cumulative hazard parameterised by a Bernstein polynomial. The model is typically fitted with time-varying coefficients, all types of random censoring and trunction are allowed.

The responses is bounded (bounds = c(0, Inf)) when specified as a Surv object. Otherwise, bounds can be specified via ....

Value

An object of class Aareg, with corresponding coef, vcov, logLik, estfun, summary, print, plot and predict methods.

References

Torsten Hothorn, Lisa Moest, Peter Buehlmann (2018), Most Likely Transformations, *Scandinavian Journal of Statistics*, **45**(1), 110–134, doi:10.1111/sjos.12291.

Examples

```
data("GBSG2", package = "TH.data")
library("survival")
GBSG2$time <- as.numeric(GBSG2$time)
GBSG2$y <- with(GBSG2, Surv(time, cens))

### Cox proportional hazards model
m1 <- Coxph(y ~ horTh, data = GBSG2, support = c(1, 1500))
logLik(m1)

### Aalen additive hazards model with time-varying effects
m2 <- Aareg(y | horTh ~ 1, data = GBSG2, support = c(1, 1500))
logLik(m2)

### compare the hazard functions
nd <- data.frame(horTh = unique(GBSG2$horTh))
col <- 1:2
lty <- 1:2
plot(as.mlt(m1), newdata = nd, type = "hazard",
     col = col, lty = lty[1], xlab = "time")
plot(as.mlt(m2), newdata = nd, type = "hazard",
     col = col, lty = 2, add = TRUE)
```

```
legend("topright", col = rep(col, each = 2),
       lty = rep(1:2), bty = "n",
       legend = paste(rep(paste("horTh:",
                                levels(nd$horTh)), each = 2),
                      rep(c("Cox", "Aalen"), 2)))
```

BoxCox *(Similar to) Box-Cox Models*

Description

Non-normal linear regression inspired by Box-Cox models

Usage

```
BoxCox(formula, data, subset, weights, offset, cluster, na.action = na.omit, ...)
```

Arguments

formula	an object of class "formula": a symbolic description of the model structure to be fitted. The details of model specification are given under tram and in the package vignette.
data	an optional data frame, list or environment (or object coercible by as.data.frame to a data frame) containing the variables in the model. If not found in data, the variables are taken from environment(formula).
subset	an optional vector specifying a subset of observations to be used in the fitting process.
weights	an optional vector of weights to be used in the fitting process. Should be NULL or a numeric vector. If present, the weighted log-likelihood is maximised.
offset	this can be used to specify an _a priori_ known component to be included in the linear predictor during fitting. This should be NULL or a numeric vector of length equal to the number of cases.
cluster	optional factor with a cluster ID employed for computing clustered covariances.
na.action	a function which indicates what should happen when the data contain NAs. The default is set to na.omit.
...	additional arguments to tram.

Details

A normal model for transformed responses, where the transformation is estimated from the data simultaneously with the regression coefficients. This is similar to a Box-Cox transformation, but the technical details differ. Examples can be found in the package vignette.

The model is defined with a negative shift term. Large values of the linear predictor correspond to large values of the conditional expectation response (but this relationship is potentially nonlinear).

Value

An object of class BoxCox, with corresponding coef, vcov, logLik, estfun, summary, print, plot and predict methods.

References

Torsten Hothorn, Lisa Moest, Peter Buehlmann (2018), Most Likely Transformations, *Scandinavian Journal of Statistics*, **45**(1), 110–134, doi:10.1111/sjos.12291.

Examples

```
data("BostonHousing2", package = "mlbench")

lm(cmedv ~ crim + zn + indus + chas + nox + rm + age + dis +
           rad + tax + ptratio + b + lstat, data = BostonHousing2)

BoxCox(cmedv ~ chas + crim + zn + indus + nox +
               rm + age + dis + rad + tax + ptratio + b + lstat,
               data = BostonHousing2)
```

Colr *Continuous Outcome Logistic Regression*

Description

A proportional-odds model for continuous variables

Usage

```
Colr(formula, data, subset, weights, offset, cluster, na.action = na.omit, ...)
```

Arguments

formula	an object of class "formula": a symbolic description of the model structure to be fitted. The details of model specification are given under tram and in the package vignette.
data	an optional data frame, list or environment (or object coercible by as.data.frame to a data frame) containing the variables in the model. If not found in data, the variables are taken from environment(formula).
subset	an optional vector specifying a subset of observations to be used in the fitting process.
weights	an optional vector of weights to be used in the fitting process. Should be NULL or a numeric vector. If present, the weighted log-likelihood is maximised.
offset	this can be used to specify an _a priori_ known component to be included in the linear predictor during fitting. This should be NULL or a numeric vector of length equal to the number of cases.

cluster	optional factor with a cluster ID employed for computing clustered covariances.
na.action	a function which indicates what should happen when the data contain NAs. The default is set by the na.action setting of options, and is na.fail if that is unset.
...	additional arguments to tram.

Details

Simultaneous estimation of all possible binary logistic models obtained by dichotomisation of a continuous response. The regression coefficients can be constant allowing for an interpretation as log-odds ratios.

The model is defined with a positive shift term, thus exp(coef()) is the multiplicative change of the odds ratio (conditional odds of treatment or for a one unit increase in a numeric variable divided by conditional odds of reference). Large values of the linear predictor correspond to small values of the conditional expectation response (but this relationship is nonlinear).

Value

An object of class Colr, with corresponding coef, vcov, logLik, estfun, summary, print, plot and predict methods.

References

Tina Lohse, Sabine Rohrmann, David Faeh and Torsten Hothorn (2017), Continuous Outcome Logistic Regression for Analyzing Body Mass Index Distributions, *F1000Research*, **6**(1933), doi:10.12688/f1000research.12934.1.

Torsten Hothorn, Lisa Moest, Peter Buehlmann (2018), Most Likely Transformations, *Scandinavian Journal of Statistics*, **45**(1), 110–134, doi:10.1111/sjos.12291.

Examples

```
data("BostonHousing2", package = "mlbench")

lm(cmedv ~ crim + zn + indus + chas + nox + rm + age + dis +
           rad + tax + ptratio + b + lstat, data = BostonHousing2)

Colr(cmedv ~ chas + crim + zn + indus + nox +
             rm + age + dis + rad + tax + ptratio + b + lstat,
             data = BostonHousing2)
```

Coxph *Cox Proportional Hazards Model*

Description

Cox model with fully parameterised baseline hazard function

Usage

```
Coxph(formula, data, subset, weights, offset, cluster, na.action = na.omit, ...)
```

Arguments

formula	an object of class "formula": a symbolic description of the model structure to be fitted. The details of model specification are given under tram and in the package vignette.
data	an optional data frame, list or environment (or object coercible by as.data.frame to a data frame) containing the variables in the model. If not found in data, the variables are taken from environment(formula).
subset	an optional vector specifying a subset of observations to be used in the fitting process.
weights	an optional vector of weights to be used in the fitting process. Should be NULL or a numeric vector. If present, the weighted log-likelihood is maximised.
offset	this can be used to specify an _a priori_ known component to be included in the linear predictor during fitting. This should be NULL or a numeric vector of length equal to the number of cases.
cluster	optional factor with a cluster ID employed for computing clustered covariances.
na.action	a function which indicates what should happen when the data contain NAs. The default is set to na.omit.
...	additional arguments to tram.

Details

The original implementation of Cox models via the partial likelihood, treating the baseline hazard function as a nuisance parameter, is available in coxph. This function allows simultaneous estimation of the log-hazard ratios and the log-cumulative baseline hazard, the latter parameterised by a Bernstein polynomial. The model can be fitted under stratification (time-varying coefficients), all types of random censoring and trunction. An early reference to this parameterisation is McLain and Ghosh (2013).

The response is bounded (bounds = c(0, Inf)) when specified as a Surv object. Otherwise, bounds can be specified via ....

Parameters are log-hazard ratios comparing treatment (or a one unit increase in a numeric variable) with a reference.

Value

An object of class `Coxph`, with corresponding `coef`, `vcov`, `logLik`, `estfun`, `summary`, `print`, `plot` and `predict` methods.

References

Alexander C. McLain and Sujit K. Ghosh (2013). Efficient Sieve Maximum Likelihood Estimation of Time-Transformation Models, *Journal of Statistical Theory and Practice*, **7**(2), 285–303, doi:10.1080/15598608.2013.772835.

Torsten Hothorn, Lisa Moest, Peter Buehlmann (2018), Most Likely Transformations, *Scandinavian Journal of Statistics*, **45**(1), 110–134, doi:10.1111/sjos.12291.

Examples

```
data("GBSG2", package = "TH.data")

library("survival")
(m1 <- coxph(Surv(time, cens) ~ horTh, data = GBSG2))

(m2 <- Coxph(Surv(time, cens) ~ horTh, data = GBSG2))

### Wald intervals
confint(m1)
confint(m2)
### profile likelihood interval
confint(profile(m2))
### score interval
confint(score_test(m2))
### permutation score interval; uses permutation distribution
### see coin::independence_test
## Not run: confint(perm_test(m2))
```

Lehmann *Proportional Reverse Time Hazards Linear Regression*

Description

Non-normal linear regression for Lehmann-alternatives

Usage

```
Lehmann(formula, data, subset, weights, offset, cluster, na.action = na.omit, ...)
```

Arguments

formula	an object of class "formula": a symbolic description of the model structure to be fitted. The details of model specification are given under tram and in the package vignette.
data	an optional data frame, list or environment (or object coercible by as.data.frame to a data frame) containing the variables in the model. If not found in data, the variables are taken from environment(formula).
subset	an optional vector specifying a subset of observations to be used in the fitting process.
weights	an optional vector of weights to be used in the fitting process. Should be NULL or a numeric vector. If present, the weighted log-likelihood is maximised.
offset	this can be used to specify an _a priori_ known component to be included in the linear predictor during fitting. This should be NULL or a numeric vector of length equal to the number of cases.
cluster	optional factor with a cluster ID employed for computing clustered covariances.
na.action	a function which indicates what should happen when the data contain NAs. The default is set to na.omit.
...	additional arguments to tram.

Details

This transformation model uses the cumulative distribution function for the standard Gumbel maximum extreme value distribution to map the shifted transformation function into probabilities. The exponential of the shift paramater can be interpreted as a Lehmann-alternative or reverse time hazard ratio.

Value

An object of class Lehmann, with corresponding coef, vcov, logLik, estfun, summary, print, plot and predict methods.

References

Erich L. Lehmann (1953), The Power of Rank Tests, *The Annals of Mathematical Statistics*, **24**(1), 23-43.

Torsten Hothorn, Lisa Moest, Peter Buehlmann (2018), Most Likely Transformations, *Scandinavian Journal of Statistics*, **45**(1), 110–134, doi:10.1111/sjos.12291.

Examples

```
data("BostonHousing2", package = "mlbench")

lm(cmedv ~ crim + zn + indus + chas + nox + rm + age + dis +
           rad + tax + ptratio + b + lstat, data = BostonHousing2)

Lehmann(cmedv ~ chas + crim + zn + indus + nox +
```

```
rm + age + dis + rad + tax + ptratio + b + lstat,
data = BostonHousing2)
```

Lm *Normal Linear Model*

Description

Normal linear model with benefits

Usage

```
Lm(formula, data, subset, weights, offset, cluster, na.action = na.omit, ...)
```

Arguments

formula	an object of class "formula": a symbolic description of the model structure to be fitted. The details of model specification are given under tram and in the package vignette.
data	an optional data frame, list or environment (or object coercible by as.data.frame to a data frame) containing the variables in the model. If not found in data, the variables are taken from environment(formula).
subset	an optional vector specifying a subset of observations to be used in the fitting process.
weights	an optional vector of weights to be used in the fitting process. Should be NULL or a numeric vector. If present, the weighted log-likelihood is maximised.
offset	this can be used to specify an _a priori_ known component to be included in the linear predictor during fitting. This should be NULL or a numeric vector of length equal to the number of cases.
cluster	optional factor with a cluster ID employed for computing clustered covariances.
na.action	a function which indicates what should happen when the data contain NAs. The default is set to na.omit.
...	additional arguments to tram.

Details

A normal linear model with simulaneous estimation of regression coefficients and scale parameter(s). This function also allows for stratum-specific intercepts and variances as well as censoring and truncation in the response.

Note that the scale of the parameters is different from what is reported by lm; the discrepancies are explained in the package vignette.

The model is defined with a negative shift term. Large values of the linear predictor correspond to large values of the conditional expectation response.

Value

An object of class `Lm`, with corresponding `coef`, `vcov`, `logLik`, `estfun`, `summary`, `print`, `plot` and `predict` methods.

References

Torsten Hothorn, Lisa Moest, Peter Buehlmann (2018), Most Likely Transformations, *Scandinavian Journal of Statistics*, **45**(1), 110–134, doi:10.1111/sjos.12291.

Examples

```
data("BostonHousing2", package = "mlbench")

lm(cmedv ~ crim + zn + indus + chas + nox + rm + age + dis +
           rad + tax + ptratio + b + lstat, data = BostonHousing2)

Lm(cmedv ~ chas + crim + zn + indus + nox +
           rm + age + dis + rad + tax + ptratio + b + lstat,
           data = BostonHousing2)
```

`mmlt` *Multivariate Conditional Transformation Models*

Description

A proof-of-concept implementation of multivariate conditional transformation models

Usage

```
mmlt(..., formula = ~ 1, data, theta = NULL,
     control.outer = list(trace = FALSE), scale = FALSE, dofit = TRUE)
```

Arguments

`...`	marginal transformation models, one for each response
`formula`	a model formula describing a model for the dependency structure via the lambda parameters. The default is set to `~ 1` for constant lambdas.
`data`	a data.frame
`theta`	an optional vector of starting values
`control.outer`	a list controlling `auglag`
`scale`	logical; parameters are not scaled prior to optimisation by default
`dofit`	logical; parameters are fitted by default, otherwise a list with log-likelihood and score function is returned

Details

The function implements multivariate conditional transformation models as described by Klein et al (2020). The response is assumed absolutely continuous at the moment, discrete versions will be added later.

Below is a simple example for an unconditional bivariate distribution. See `demo("undernutrition", package = "tram")` for a conditional three-variate example.

Value

An object of class `mmlt` with `coef` and `predict` methods.

References

Nadja Klein, Torsten Hothorn, Luisa Barbanti, Thomas Kneib (2020), Multivariate Conditional Transformation Models. *Scandinavian Journal of Statistics*, doi:10.1111/sjos.12501.

Examples

```
data("cars")

### fit unconditional bivariate distribution of speed and distance to stop
## fit unconditional marginal transformation models
m_speed <- BoxCox(speed ~ 1, data = cars, support = ss <- c(4, 25),
                  add = c(-5, 5))
m_dist <- BoxCox(dist ~ 1, data = cars, support = sd <- c(0, 120),
                 add = c(-5, 5))

## fit multivariate unconditional transformation model
m_speed_dist <- mmlt(m_speed, m_dist, formula = ~ 1, data = cars)

## lambda defining the Cholesky of the precision matrix,
## with standard error
coef(m_speed_dist, newdata = cars[1,], type = "Lambda")
sqrt(vcov(m_speed_dist)["dist.sped.(Intercept)",
                        "dist.sped.(Intercept)"])

## linear correlation, ie Pearson correlation of speed and dist after
## transformation to bivariate normality
(r <- coef(m_speed_dist, newdata = cars[1,], type = "Corr"))

## Spearman's rho (rank correlation), can be computed easily
## for Gaussian copula as
(rs <- 6 * asin(r / 2) / pi)

## evaluate joint and marginal densities (needs to be more user-friendly)
nd <- expand.grid(c(nd_s <- mkgrid(m_speed, 100), nd_d <- mkgrid(m_dist, 100)))
nd$hs <- predict(m_speed_dist, newdata = nd, marginal = 1L)
nd$hps <- predict(m_speed_dist, newdata = nd, marginal = 1L,
                  deriv = c("speed" = 1))
nd$hd <- predict(m_speed_dist, newdata = nd, marginal = 2L)
```

```
nd$hpd <- predict(m_speed_dist, newdata = nd, marginal = 2L,
                  deriv = c("dist" = 1))

## joint density
nd$d <- with(nd,
             dnorm(hs) *
             dnorm(coef(m_speed_dist)["dist.sped.(Intercept)"] * hs + hd) *
             hps * hpd)

## compute marginal densities
nd_s <- as.data.frame(nd_s)
nd_s$d <- predict(m_speed_dist, newdata = nd_s, type = "density")
nd_d <- as.data.frame(nd_d)
nd_d$d <- predict(m_speed_dist, newdata = nd_d, marginal = 2L,
                  type = "density")

## plot bivariate and marginal distribution
col1 <- rgb(.1, .1, .1, .9)
col2 <- rgb(.1, .1, .1, .5)
w <- c(.8, .2)
layout(matrix(c(2, 1, 4, 3), nrow = 2), width = w, height = rev(w))
par(mai = c(1, 1, 0, 0) * par("mai"))
sp <- unique(nd$speed)
di <- unique(nd$dist)
d <- matrix(nd$d, nrow = length(sp))
contour(sp, di, d, xlab = "Speed (in mph)", ylab = "Distance (in ft)", xlim = ss, ylim = sd,
        col = col1)
points(cars$speed, cars$dist, pch = 19, col = col2)
mai <- par("mai")
par(mai = c(0, 1, 0, 1) * mai)
plot(d ~ speed, data = nd_s, xlim = ss, type = "n", axes = FALSE,
     xlab = "", ylab = "")
polygon(nd_s$speed, nd_s$d, col = col2, border = FALSE)
par(mai = c(1, 0, 1, 0) * mai)
plot(dist ~ d, data = nd_d, ylim = sd, type = "n", axes = FALSE,
     xlab = "", ylab = "")
polygon(nd_d$d, nd_d$dist, col = col2, border = FALSE)

### NOTE: marginal densities are NOT normal, nor is the joint
### distribution. The non-normal shape comes from the data-driven
### transformation of both variables to joint normality in this model.
```

---

mtram *Transformation Models for Clustered Data*

Description

Marginally interpretable transformation models for clustered data. Highly experimental, use at your own risk.

Usage

```
mtram(object, formula, data,
      grd = SparseGrid::createSparseGrid(type = "KPU",
                dimension = length(rt$cnms[[1]]), k = 10),
      Hessian = FALSE,  tol = .Machine$double.eps, ...)
```

Arguments

object	A tram object.
formula	A formula specifying the random effects.
data	A data frame.
grd	A sparse grid used for numerical integration to get the likelihood.
Hessian	A logical, if TRUE, the hessian is computed and returned.
tol	numerical tolerance.
...	Additional argument.

Details

A Gaussian copula with a correlation structure obtained from a random intercept or random intercept / random slope model (that is, clustered or longitudinal data can by modelled only) is used to capture the correlations whereas the marginal distributions are described by a transformation model. The methodology is described in Hothorn (2019) and examples are given in the mtram package vignette.

This is a proof-of-concept implementation and still highly experimental. Only coef() and logLik() methods are available at the moment.

Value

An object of class tram with coef() and logLik() methods.

References

Torsten Hothorn (2019). Marginally Interpretable Parametric Linear Transformation Models for Clustered Observations. Technical Report.

---

perm_test *Permutation Transformation Tests*

Description

P-values for a parameter in a linear transformation model and corresponding confidence intervals obtained from by the permutation principle

Usage

```
perm_test(object, ...)
## S3 method for class 'tram'
perm_test(object, parm = names(coef(object)),
    statistic = c("Score", "Likelihood", "Wald"),
    alternative = c("two.sided", "less", "greater"),
    nullvalue = 0, confint = TRUE, level = .95,
    Taylor = FALSE, block_permutation = TRUE, maxsteps = 25, ...)
```

Arguments

`object`	an object of class `tram`
`parm`	a vector of names of parameters to be tested. These parameters must be present in `object`.
`statistic`	a character string specifying the statistic to be permuted. The default `Score` is the classical permutation test for the esiduals of a model excluding the parameter `parm`. Only available for `nullvalue = 0`, confidence intervals are not available. Permuting the likelihood or the model coefficients under the nullvalue is highly expermimental as are the corresponding confidence intervals.
`alternative`	a character string specifying the alternative hypothesis, must be one of `"two.sided"` (default), `"greater"` or `"less"`.
`nullvalue`	a number specifying an optional parameter used to form the null hypothesis.
`confint`	a logical indicating whether a confidence interval should be computed. Score confidence intervals are computed by default. A 1st order Taylor approximation to the Score statistc is used with `Taylor = TRUE` (in case numerical inversion of the score statistic fails, Wald-type confidence intervals relying from this approximation are returned) . For the remaining likelihood and Wald statistics, confidence intervals are highly experimental (and probably not worth looking at).
`level`	the confidence level.
`block_permutation`	a logical indicating wheather stratifying variables shall be interpreted as blocks defining admissible permutations.
`Taylor`	a logical requesting the use of a 1st order Taylor approximation when inverting the score statistic.
`maxsteps`	number of function evaluations when inverting the score statistic for computing confidence intervals.
`...`	additional arguments to `independence_test`.

Details

Permutation test for one single parameters in the linear predictor of `object` is computed. This parameters must be present in `object`. This is somewhat experimental and not recommended for serious practical use (yet!).

Value

An object of class `htest` or a list thereof. See `Coxph` for an example.

Examples

```
## Tritiated Water Diffusion Across Human Chorioamnion
## Hollander and Wolfe (1999, p. 110, Tab. 4.1)
diffusion <- data.frame(
    pd = c(0.80, 0.83, 1.89, 1.04, 1.45, 1.38, 1.91, 1.64, 0.73, 1.46,
           1.15, 0.88, 0.90, 0.74, 1.21),
    age = factor(rep(c("At term", "12-26 Weeks"), c(10, 5)))
)

### plot the two quantile functions
boxplot(pd ~ age, data = diffusion)

### the Wilcoxon rank sum test, with a confidence interval
### for a median shift
wilcox.test(pd ~ age, data = diffusion, conf.int = TRUE, exact = TRUE)

### a corresponding parametric transformation model with a log-odds ratio
### difference parameter, ie a difference on the log-odds scale
md <- Colr(pd ~ age, data = diffusion)

### assess model fit by plotting estimated distribution fcts
agef <- sort(unique(diffusion$age))
col <- c("black", "darkred")
plot(as.mlt(md), newdata = data.frame(age = agef),
     type = "distribution", col = col)
legend("bottomright", col = col, lty = 1, legend = levels(agef),
       bty = "n", pch = 19)
## compare with ECDFs: not too bad (but not good, either)
npfit <- with(diffusion, tapply(pd, age, ecdf))
lines(npfit[[1]], col = col[1])
lines(npfit[[2]], col = col[2])

### Wald confidence interval
confint(md)

### Likelihood confidence interval
confint(profile(md))

### Score confidence interval
confint(score_test(md))
confint(score_test(md, Taylor = TRUE))

### exact permutation score test
(pt <- perm_test(md, confint = TRUE, distribution = "exact"))
(pt <- perm_test(md, confint = TRUE, distribution = "exact",
                 Taylor = TRUE))
```

```
### compare with probabilistic indices obtained from asht::wmwTest
if (require("asht", warn.conflicts = FALSE)) {
    print(wt2 <- wmwTest(pd ~ I(relevel(age, "At term")),
                    data = diffusion, method = "exact.ce"))
    ### as log-odds ratios
    print(PI(prob = wt2$conf.int))
    print(PI(prob = wt2$estimate))
}
```

Polr *Ordered Categorical Regression*

Description

Some regression models for ordered categorical responses

Usage

```
Polr(formula, data, subset, weights, offset, cluster, na.action = na.omit,
     method = c("logistic", "probit", "loglog", "cloglog"), ...)
```

Arguments

formula	an object of class "formula": a symbolic description of the model structure to be fitted. The details of model specification are given under tram and in the package vignette.
data	an optional data frame, list or environment (or object coercible by as.data.frame to a data frame) containing the variables in the model. If not found in data, the variables are taken from environment(formula).
subset	an optional vector specifying a subset of observations to be used in the fitting process.
weights	an optional vector of weights to be used in the fitting process. Should be NULL or a numeric vector. If present, the weighted log-likelihood is maximised.
offset	this can be used to specify an _a priori_ known component to be included in the linear predictor during fitting. This should be NULL or a numeric vector of length equal to the number of cases.
cluster	optional factor with a cluster ID employed for computing clustered covariances.
na.action	a function which indicates what should happen when the data contain NAs. The default is set by the na.action setting of options, and is na.fail if that is unset.
method	a character describing the link function.
...	additional arguments to tram.

Details

Models for ordered categorical responses reusing the interface of `polr`. Allows for stratification, censoring and trunction.

The model is defined with a negative shift term, thus `exp(coef())` is the multiplicative change of the odds ratio (conditional odds for reference divided by conditional odds of treatment or for a one unit increase in a numeric variable). Large values of the linear predictor correspond to large values of the conditional expectation response (but this relationship is nonlinear).

Value

An object of class `Polr`, with corresponding `coef`, `vcov`, `logLik`, `estfun`, `summary`, `print`, `plot` and `predict` methods.

References

Torsten Hothorn, Lisa Moest, Peter Buehlmann (2018), Most Likely Transformations, *Scandinavian Journal of Statistics*, **45**(1), 110–134, doi:10.1111/sjos.12291.

Examples

```
data("wine", package = "ordinal")

library("MASS")
polr(rating ~ temp + contact, data = wine)

Polr(rating ~ temp + contact, data = wine)
```

`score_test` *Transformation Score Tests and Confidence Intervals*

Description

P-values and confidence intervals for parameters in linear transformation models obtained from by the score test principle

Usage

```
score_test(object, ...)
## S3 method for class 'tram'
score_test(object, parm = names(coef(object)),
    alternative = c("two.sided", "less", "greater"), nullvalue = 0,
    confint = TRUE, level = .95, Taylor = FALSE, maxsteps = 25, ...)
```

Arguments

object	an object of class tram
parm	a vector of names of parameters to be tested. These parameters must be present in object.
alternative	a character string specifying the alternative hypothesis, must be one of "two.sided" (default), "greater" or "less".
nullvalue	a number specifying an optional parameter used to form the null hypothesis.
confint	a logical indicating whether a confidence interval should be computed. Score confidence intervals are computed by default. A 1st order Taylor approximation to the Score statistc is used with Taylor = TRUE (in case numerical inversion of the score statistic fails, Wald confidence intervals relying from this approximation are returned).
level	the confidence level.
Taylor	a logical requesting the use of a 1st order Taylor approximation when inverting the score statistic.
maxsteps	number of function evaluations when inverting the score statistic for computing confidence intervals.
...	additional arguments, currently ignored.

Details

Score tests and confidence intervals for the parameters in the linear predictor of object are computed. These parameters must be present in object.

Value

An object of class htest or a list thereof. See Coxph for an example. A corresponding permutation test for parameters in a transformation models is available in perm_test.

Survreg *Parametric Survival Models*

Description

Weibull, log-normal, log-logistic and other parametric models (not exclusively) for survival analysis

Usage

```
Survreg(formula, data, subset, weights, offset, cluster, na.action = na.omit,
        dist = c("weibull", "logistic", "gaussian", "exponential", "rayleigh",
                 "loggaussian", "lognormal", "loglogistic"), scale = 0, ...)
```

Arguments

formula	an object of class "formula": a symbolic description of the model structure to be fitted. The details of model specification are given under tram and in the package vignette.
data	an optional data frame, list or environment (or object coercible by as.data.frame to a data frame) containing the variables in the model. If not found in data, the variables are taken from environment(formula).
subset	an optional vector specifying a subset of observations to be used in the fitting process.
weights	an optional vector of weights to be used in the fitting process. Should be NULL or a numeric vector. If present, the weighted log-likelihood is maximised.
offset	this can be used to specify an _a priori_ known component to be included in the linear predictor during fitting. This should be NULL or a numeric vector of length equal to the number of cases.
cluster	optional factor with a cluster ID employed for computing clustered covariances.
na.action	a function which indicates what should happen when the data contain NAs. The default is set by the na.action setting of options, and is na.fail if that is unset.
dist	character defining the conditional distribution of the (not necessarily positive) response, current choices include Weibull, logistic, normal, exponential, Rayleigh, log-normal (same as log-gaussian), or log-logistic.
scale	a fixed value for the scale parameter(s).
...	additional arguments to tram.

Details

Parametric survival models reusing the interface of survreg. The parameterisation is, however, a little different, see the package vignette.

The model is defined with a negative shift term. Large values of the linear predictor correspond to large values of the conditional expectation response (but this relationship is nonlinear). Parameters are log-hazard ratios comparing a reference with treatment (or a one unit increase in a numeric variable).

Value

An object of class Survreg, with corresponding coef, vcov, logLik, estfun, summary, print, plot and predict methods.

References

Torsten Hothorn, Lisa Moest, Peter Buehlmann (2018), Most Likely Transformations, *Scandinavian Journal of Statistics*, **45**(1), 110–134, doi:10.1111/sjos.12291.

Examples

```
data("GBSG2", package = "TH.data")

library("survival")
survreg(Surv(time, cens) ~ horTh, data = GBSG2)

Survreg(Surv(time, cens) ~ horTh, data = GBSG2)
```

`tram` *Stratified Linear Transformation Models*

Description

Likelihood-inference for stratified linear transformation models

Usage

```
tram(formula, data, subset, weights, offset, cluster, na.action = na.omit,
   distribution = c("Normal", "Logistic", "MinExtrVal", "MaxExtrVal", "Exponential"),
     transformation = c("discrete", "linear", "logarithmic", "smooth"),
     LRtest = TRUE, prob = c(0.1, 0.9), support = NULL,
     bounds = NULL, add = c(0, 0), order = 6,
     negative = TRUE, scale = TRUE, scale_shift = FALSE, extrapolate = FALSE,
     log_first = FALSE, sparse_nlevels = Inf,
     model_only = FALSE, constraints = NULL, ...)
tram_data(formula, data, subset, weights, offset, cluster, na.action = na.omit)
```

Arguments

`formula`	an object of class `"formula"`: a symbolic description of the model structure to be fitted. The details of model specification are given under Details and in the package vignette.
`data`	an optional data frame, list or environment (or object coercible by `as.data.frame` to a data frame) containing the variables in the model. If not found in `data`, the variables are taken from `environment(formula)`.
`subset`	an optional vector specifying a subset of observations to be used in the fitting process.
`weights`	an optional vector of case weights to be used in the fitting process. Should be `NULL` or a numeric vector. If present, the weighted log-likelihood is maximised.
`offset`	this can be used to specify an _a priori_ known component to be included in the linear predictor during fitting. This should be `NULL` or a numeric vector of length equal to the number of cases.
`cluster`	optional factor with a cluster ID employed for computing clustered covariances.

na.action	a function which indicates what should happen when the data contain NAs. The default is set to na.omit.
distribution	character specifying how the transformation function is mapped into probabilities. Available choices include the cumulative distribution functions of the standard normal, the standard logistic and the standard minimum extreme value distribution.
transformation	character specifying the complexity of the response-transformation. For discrete responses, one parameter is assigned to each level (except the last one), for continuous responses linear, log-linear and smooth (parameterised as a Bernstein polynomial) function are implemented.
LRtest	logical specifying if a likelihood-ratio test for the null of all coefficients in the linear predictor being zero shall be performed.
prob	two probabilities giving quantiles of the response defining the support of a smooth Bernstein polynomial (if transformation = "smooth").
support	a vector of two elements; the support of a smooth Bernstein polynomial (if transformation = "smooth").
bounds	an interval defining the bounds of a real sample space.
add	add these values to the support before generating a grid via mkgrid.
order	integer >= 1 defining the order of the Bernstein polynomial (if transformation = "smooth").
negative	logical defining the sign of the linear predictor.
scale	logical defining if variables in the linear predictor shall be scaled. Scaling is internally used for model estimation, rescaled coefficients are reported in model output.
scale_shift	a logical choosing between two different model types in the presence of a scaling term, see ctm.
extrapolate	logical defining the behaviour of the Bernstein transformation function outside support. The default FALSE is to extrapolate linearily without requiring the second derivative of the transformation function to be zero at support. If TRUE, this additional constraint is respected.
sparse_nlevels	integer; use a sparse model matrix if the number of levels of an ordered factor is at least as large as sparse_nlevels.
log_first	logical; if TRUE, a Bernstein polynomial is defined on the log-scale.
model_only	logical, if TRUE the unfitted model is returned.
constraints	additional constraints on regression coefficients in the linear predictor of the form lhs %*% coef(object) >= rhs, where lhs and rhs can be specified as a character (as in glht) or by a matrix lhs (assuming rhs = 0), or as a list containing the two elements lhs and rhs.
...	additional arguments.

Details

The model formula is of the form y | s ~ x | z where y is an at least ordered response variable, s are the variables defining strata and x defines the linear predictor. Optionally, z defines a scaling

term (see ctm). y ~ x defines a model without strata (but response-varying intercept function) and y | s ~ 0 sets-up response-varying coefficients for all variables in s.

The two functions tram and tram_data are not intended to be called directly by users. Instead, functions Coxph (Cox proportional hazards models), Survreg (parametric survival models), Polr (models for ordered categorical responses), Lm (normal linear models), BoxCox (non-normal linear models) or Colr (continuous outcome logistic regression) allow direct access to the corresponding models.

The model class and the specific models implemented in **tram** are explained in the package vignette of package **tram**. The underlying theory of most likely transformations is presented in Hothorn et al. (2018), computational and modelling aspects in more complex situations are discussed by Hothorn (2018).

Value

An object of class tram inheriting from mlt.

References

Torsten Hothorn, Lisa Moest, Peter Buehlmann (2018), Most Likely Transformations, *Scandinavian Journal of Statistics*, **45**(1), 110–134, doi:10.1111/sjos.12291.

Torsten Hothorn (2018), Most Likely Transformations: The mlt Package, *Journal of Statistical Software*, forthcoming. URL: https://cran.r-project.org/package=mlt.docreg

Examples

```
data("BostonHousing2", package = "mlbench")

### unconstrained regression coefficients
### BoxCox calls tram internally
m1 <- BoxCox(cmedv ~ chas + crim + zn + indus + nox +
             rm + age + dis + rad + tax + ptratio + b + lstat,
             data = BostonHousing2)

### now with two constraints on regression coefficients
m2 <- BoxCox(cmedv ~ chas + crim + zn + indus + nox +
             rm + age + dis + rad + tax + ptratio + b + lstat,
             data = BostonHousing2,
             constraints = c("crim >= 0", "chas1 + rm >= 1.5"))
coef(m1)
coef(m2)

K <- matrix(0, nrow = 2, ncol = length(coef(m2)))
colnames(K) <- names(coef(m2))
K[1, "crim"] <- 1
K[2, c("chas1", "rm")] <- 1
m3 <- BoxCox(cmedv ~ chas + crim + zn + indus + nox +
             rm + age + dis + rad + tax + ptratio + b + lstat,
             data = BostonHousing2,
             constraints = list(K, c(0, 1.5)))
```

```
all.equal(coef(m2), coef(m3))
```

---

tram-methods *Methods for Stratified Linear Transformation Models*

Description

Methods for objects inheriting from class tram

Usage

```
## S3 method for class 'tram'
as.mlt(object)
## S3 method for class 'tram'
model.frame(formula, ...)
## S3 method for class 'tram'
model.matrix(object, data = object$data, with_baseline = FALSE, ...)
## S3 method for class 'stram'
model.matrix(object, data = object$data, with_baseline = FALSE,
       what = c("shifting", "scaling"), ...)
## S3 method for class 'tram'
coef(object, with_baseline = FALSE, ...)
## S3 method for class 'Lm'
coef(object, as.lm = FALSE, ...)
## S3 method for class 'Survreg'
coef(object, as.survreg = FALSE, ...)
## S3 method for class 'tram'
vcov(object, with_baseline = FALSE, complete = FALSE, ...)
## S3 method for class 'tram'
logLik(object, parm = coef(as.mlt(object), fixed = FALSE), ...)
## S3 method for class 'tram'
estfun(object, parm = coef(as.mlt(object), fixed = FALSE), ...)
## S3 method for class 'tram'
predict(object, newdata = model.frame(object),
        type = c("lp", "trafo", "distribution", "survivor", "density",
                 "logdensity", "hazard", "loghazard", "cumhazard",
                 "quantile"),
           ...)
## S3 method for class 'stram'
predict(object, newdata = model.frame(object),
        type = c("lp", "trafo", "distribution", "survivor", "density",
                 "logdensity", "hazard", "loghazard", "cumhazard",
                 "quantile"), what = c("shifting", "scaling"),
           ...)
## S3 method for class 'tram'
plot(x, newdata = model.frame(x),
```

```
     which = c("QQ-PIT", "baseline only", "distribution"),
     confidence = c("none", "interval", "band"), level = 0.95,
     K = 50, cheat = K, col = "black", fill = "lightgrey", lwd = 1, ...)
## S3 method for class 'tram'
residuals(object, ...)
## S3 method for class 'tram'
PI(object, newdata = model.frame(object), reference = 0,
                  one2one = FALSE, ...)
## Default S3 method:
PI(object, prob, link = "logistic", ...)
## S3 method for class 'tram'
OVL(object, newdata = model.frame(object), reference = 0,
                  one2one = FALSE, ...)
## Default S3 method:
OVL(object, link = "logistic", ...)
## S3 method for class 'tram'
TV(object, newdata = model.frame(object), reference = 0,
                  one2one = FALSE, ...)
## Default S3 method:
TV(object, link = "logistic", ...)
## S3 method for class 'tram'
L1(object, newdata = model.frame(object), reference = 0,
                  one2one = FALSE, ...)
## Default S3 method:
L1(object, link = "logistic", ...)
## S3 method for class 'tram'
ROC(object, newdata = model.frame(object), reference = 0,
                   prob = 1:99 / 100, one2one = FALSE, ...)
## Default S3 method:
ROC(object, prob = 1:99 / 100, link = "logistic", ...)
## S3 method for class 'ROCtram'
plot(x, lty = 1:ncol(x), col = "black",
     fill = "lightgrey", lwd = 1, ...)
```

Arguments

`object, formula, x`	a fitted stratified linear transformation model inheriting from class `tram`. `PI` also takes a numeric vector in the default method.
`data`	an optional data frame.
`with_baseline`	logical, if `TRUE` all model parameters are returned, otherwise parameters describing the baseline transformation are ignored.
`as.lm`	logical, return parameters in the `lm` parameterisation if `TRUE`.
`as.survreg`	logical, return parameters in the `survreg` parameterisation in `TRUE`.
`parm`	model parameters, including baseline parameters.
`complete`	currently ignored
`newdata`	an optional data frame of new observations.

reference	an optional data frame of reference observations, or a numeric vector of reference values.
type	type of prediction, current options include linear predictors ("lp", of x variables in the formula y | s ~ x), transformation functions ("trafo") or distribution functions on the scale of the cdf ("distribution"), survivor function, density function, log-density function, hazard function, log-hazard function, cumulative hazard function or quantile function.
which	type of plot, either a QQ plot of the probability-integral transformed observations ("QQ-PIT"), of the baseline transformation of the whole distribution.
what	type of model matrix / linear predictor: shifting returns model model matrix / linear predictor for shift term, scaling for the scale term.
confidence	type of uncertainty assessment.
level	confidence level.
K	number of grid points in the response, see plot.ctm.
cheat	reduced number of grid points for the computation of confidence bands, see confband.
col	line color.
fill	fill color.
lwd	line width.
lty	line type.
prob	a numeric vector of probabilities..
link	a character identifying a link function.
one2one	logical, compute the ROC curve (and derived measures) comparing each row in newdata with each row in reference (FALSE, the default), or compare observations rowwise (TRUE).
...	additional arguments to the underlying methods for class mlt, see mlt-methods.

Details

coef can be used to get (and set) model parameters, logLik evaluates the log-likelihood (also for parameters other than the maximum likelihood estimate); vcov returns the estimated variance-covariance matrix (possibly taking cluster into account) and and estfun gives the score contribution by each observation. predict and plot can be used to inspect the model on different scales.

PI computes the probabilistic index (or concordance probability or AUC) for all observations in newdata, relative to reference, ie the probability

$$P(Y_1 \leq Y_0 \mid x_0, x_1)$$

of observing a smaller value of a randomly sampled observation conditional on x_1 compared to a randomly sampled reference observation, which is conditional on x_0. This is equivalent to the area under the receiver operating curve (ROC). The probability only applies within strata, response-varying coefficients are not allowed.

Under the same setup, OVL gives the overlap coefficient, which is one minus the total variation and one minus half the L_1 distance between the two conditional densities. The overlap coefficient is identical to the Youden index and the Smirnov statistic.

PI and friends also accept an argument conf.level which triggers computation of simultaneous Wald confidence intervals for these measures. Arguments in ... are forwarded to glht.

References

Torsten Hothorn, Lisa Moest, Peter Buehlmann (2018), Most Likely Transformations, *Scandinavian Journal of Statistics*, **45**(1), 110–134, doi:10.1111/sjos.12291.

See Also

mlt-methods, plot.ctm

Examples

```
data("BostonHousing2", package = "mlbench")

### fit non-normal Box-Cox type linear model with two
### baseline functions (for houses near and off Charles River)
BC_BH_2 <- BoxCox(cmedv | 0 + chas ~ crim + zn + indus + nox +
                  rm + age + dis + rad + tax + ptratio + b + lstat,
                  data = BostonHousing2)
logLik(BC_BH_2)

### classical likelihood inference
summary(BC_BH_2)

### coefficients of the linear predictor
coef(BC_BH_2)

### plot linear predictor (mean of _transformed_ response)
### vs. observed values
plot(predict(BC_BH_2, type = "lp"), BostonHousing2$cmedv)

### all coefficients
coef(BC_BH_2, with_baseline = TRUE)

### compute predicted median along with 10% and 90% quantile for the first
### observations
predict(BC_BH_2, newdata = BostonHousing2[1:3,], type = "quantile",
        prob = c(.1, .5, .9))

### plot the predicted density for these observations
plot(BC_BH_2, newdata = BostonHousing2[1:3, -1],
     which = "distribution", type = "density", K = 1000)

### evaluate the two baseline transformations, with confidence intervals
nd <- model.frame(BC_BH_2)[1:2, -1]
nd$chas <- factor(c("0", "1"))
library("colorspace")
col <- diverge_hcl(2, h = c(246, 40), c = 96, l = c(65, 90))
fill <- diverge_hcl(2, h = c(246, 40), c = 96, l = c(65, 90), alpha = .3)
plot(BC_BH_2, which = "baseline only", newdata = nd, col = col,
     confidence = "interval", fill = fill, lwd = 2,
     xlab = "Median Value", ylab = expression(h[Y]))
legend("bottomright", lty = 1, col = col,
```

```
        title = "Near Charles River", legend = c("no", "yes"), bty = "n")

### cars data; with quantile functions
plot(dist ~ speed, data = cars)
m <- Colr(dist ~ speed, data = cars)
q <- predict(as.mlt(m), newdata = data.frame(speed = s <- 6:25),
             type = "quantile", prob = c(1, 5, 9) / 10)
lines(s, q[1,])
lines(s, q[2,])
lines(s, q[3,])

nd <- data.frame(speed = s <- as.double(1:5 * 5))

# Prob(dist at speed s > dist at speed 0)
# speed 0 is reference, not a good choice here
PI(m, newdata = nd)

# Prob(dist at speed s > dist at speed 15)
lp15 <- c(predict(m, newdata = data.frame(speed = 15)))
PI(m, newdata = nd, reference = lp15)
PI(m, newdata = nd, reference = nd[3,,drop = FALSE])

# Prob(dist at speed s' > dist at speed s)
PI(m, newdata = nd, reference = nd)
# essentially:
lp <- predict(m, newdata = nd)
PI(object = dist(lp))
# same, with simultaneous confidence intervals
PI(m, newdata = nd, reference = nd, conf.level = .95)

# plot ROC curves + confidence bands
# compare speed 20 and 25 to speed 15
plot(ROC(m, newdata = nd[4:5,,drop = FALSE],
         reference = nd[3,,drop = FALSE],
         conf.level = 0.95))

# Overlap of conditional densities at speed s' and s
OVL(m, newdata = nd, reference = nd)

### ROC analysis
if (require("mlbench")) {

    layout(matrix(1:4, nrow = 2))
    data("PimaIndiansDiabetes2", package = "mlbench")
    dia <- sort(unique(PimaIndiansDiabetes2$diabetes))
    nd <- data.frame(diabetes = dia,
                     age = 29, mass = 32) ### median values

    ### unconditional ROC analysis: glucose tolerance test
    m0 <- Colr(glucose ~ diabetes, data = PimaIndiansDiabetes2)
    # ROC curve + confidence band
    plot(ROC(m0, newdata = nd[2,,drop = FALSE], conf.level = .95))
    # Wald interval for AUC
```

```
    PI(m0, newdata = nd[2,,drop = FALSE], conf.level = .95)
    # score interval for AUC
    PI(-c(coef(m0), score_test(m0)$conf.int[2:1]))

    ### adjusted ROC analysis for age and mass
    m1 <- Colr(glucose ~ diabetes + age + mass, data = PimaIndiansDiabetes2)
    # ROC curve + confidence band (this is the same for all ages /
    # masses)
    plot(ROC(m1, newdata = nd[2,,drop = FALSE],
                 reference = nd[1,,drop = FALSE],
             conf.level = .95))
    # Wald interval for adjusted AUC
    PI(m1, newdata = nd[2,,drop = FALSE], reference = nd[1,,drop = FALSE],
       conf.level = .95)
    # Score interval for adjusted AUC
    PI(-c(coef(m1)[1], score_test(m1, names(coef(m1))[1])$conf.int[2:1]))

    ### conditional ROC analysis: AUC regression ~ age + mass
    m2 <- Colr(glucose ~ diabetes * (age + mass), data = PimaIndiansDiabetes2)
    # ROC curve for a person with age = 29 and mass = 32
    plot(ROC(m2, newdata = nd[2,,drop = FALSE],
                 reference = nd[1,,drop = FALSE],
             conf.level = .95))
    # AUC for persons ages 21:81, all with mass = 32
    nd1 <- data.frame(diabetes = nd[1,"diabetes"], age = 21:81, mass = 32)
    nd2 <- data.frame(diabetes = nd[2,"diabetes"], age = 21:81, mass = 32)
    auc <- PI(m2, newdata = nd2, reference = nd1, one2one = TRUE,
              conf.level = 0.95)
    plot(nd1$age, auc[, "Estimate"], xlab = "Age (in years)", ylab =
         "AUC", ylim = c(0, 1), type = "l")
    lines(nd1$age, auc[, "lwr"], lty = 3)
    lines(nd1$age, auc[, "upr"], lty = 3)
}
```

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