

Package ‘SkeweDF’

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Title Optimization of Skewed Distributions with Birth-Death Processes

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Description Implementations of models which follow the Kolmogorov Birth-Death process framework and functions which utilize these Kolmogorov Birth-Death process models for analysis of skewed distribution functions.

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Author Andre Grageda [aut, cre]

Maintainer Andre Grageda <gragedaa@upstate.edu>

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calculate_label_coords

Label Coordinate Calculate Helper Function

Description

This function calculates coordinates for a plot given x and y bounds and location represented as percentage of plot area

Usage

```
calculate_label_coords(
  x_lower_bound,
  x_upper_bound,
  y_lower_bound,
  y_upper_bound,
  x_buffer = 0.5,
  y_buffer = 0.5,
  log_scale_x = FALSE,
```

```
    log_scale_y = FALSE  
  )
```

Arguments

x_lower_bound Numeric lowest value of x axis
x_upper_bound Numeric highest value of x axis
y_lower_bound Numeric lowest value of y axis
y_upper_bound Numeric highest value of y axis
x_buffer Numeric indicating location on x axis (0 - 1)
y_buffer Numeric indicating location on y axis (0 - 1)
log_scale_x Boolean indicating if x axis is log scale
log_scale_y Boolean indicating if y axis is log scale

Exponential

Exponential Distribution Function

Description

This function generates a vector of n length of the Exponential distribution with parameters a and b.

Usage

```
Exponential(n, a, b)
```

Arguments

n Length of vector to be generated.
a Parameter of the Exponential distribution function
b Parameter of the Exponential distribution function

Examples

```
Exponential(100, 10000, 0.8)
```

Generalized_Pareto *Generalized Pareto Distribution Function*

Description

Returns vector of length k of Generalized Pareto given a parameters theta, c, b , and rho

Usage

Generalized_Pareto(k, theta, c, b, rho)

Arguments

k	Length of vector to be generated
theta	Parameter of the Generalized Pareto function
c	Parameter of the Generalized Pareto function
b	Parameter of the Generalized Pareto function
rho	Parameter of the Generalized Pareto function

Generalized_Pareto_calc_P0_delta
Generalized Pareto Distribution Function P0 with defined delta

Description

Returns P0 of Generalized Pareto given a parameters theta, c, b , and rho

Usage

Generalized_Pareto_calc_P0_delta(theta, c, b, rho, delta)

Arguments

theta	Parameter of the Generalized Pareto function
c	Parameter of the Generalized Pareto function
b	Parameter of the Generalized Pareto function
rho	Parameter of the Generalized Pareto function
delta	Value of difference between iterations in order to output a result. Decreasing this parameter will increase accuracy of P0. Delta > 0

 Generalized_Pareto_calc_P0_iter

Generalized Pareto Distribution Function P0 with defined number of iterations

Description

Returns P0 of Generalized Pareto given a parameters theta, c, b , and rho

Usage

Generalized_Pareto_calc_P0_iter(theta, c, b, rho, iter)

Arguments

theta	Parameter of the Generalized Pareto function
c	Parameter of the Generalized Pareto function
b	Parameter of the Generalized Pareto function
rho	Parameter of the Generalized Pareto function
iter	Number of iterations to be performed for summation calculation. Increasing this parameter will increase accuracy of P0

Generalized_Yule

Generalized Yule Distribution Function

Description

This function generates a vector of n length of the Generalized Yule distribution with parameters rho and alpha.

Usage

Generalized_Yule(n, rho, alpha)

Arguments

n	Length of vector to be generated.
rho	Parameter of the Generalized Yule distribution function
alpha	Parameter of the Generalized Yule distribution function: $0 \leq \alpha < 1$

Examples

Generalized_Yule(100, 3, 0.1)

get_CI	<i>Get Mean Confidence Interval Function</i>
--------	--

Description

This function generates a vector of confidence interval based on mean of data.

Usage

```
get_CI(data, alpha)
```

Arguments

data	Data to get confidence interval from
alpha	Alpha for confidence interval calculation

get_median_CI	<i>Get Median Confidence Interval Function</i>
---------------	--

Description

This function generates a vector of ranked 95

Usage

```
get_median_CI(data)
```

Arguments

data	Data to get confidence interval from
------	--------------------------------------

get_p0	<i>Psi Criterion for RGHD parameter ratios</i>
--------	--

Description

This function generates the Psi Criterion goodness of fit value given an empirical distribution for the 2m-RGHD function. Parameters r and q/r ratios are given, as well as desired weight of pmf and use of the weighted right-tail cumulative distribution function.

Usage

```
get_p0(params, model_fn_name)
```

Arguments

params Vector of parameter for the model function
 model_fn_name name of function as a character vector

Examples

```
params <- c(2, 3, 0.9)
get_p0(params, 'Kolmogorov Waring')
```

global_fit_function *Global optimization of a given function given empirical data and parameter bounds*

Description

This function generates a single set of optimized parameters and Psi Criterion for a given function within specified starting parameter bounds. This function uses a modified grid search method for optimization

Usage

```
global_fit_function(
  param_bounds,
  data,
  model_fn_name,
  iter = 1,
  weighted_rt = FALSE,
  n_cores = 1,
  clust
)
```

Arguments

param_bounds A list of sequences which indicate space where parameters should be generated and fit

data Vector of observed values

model_fn_name Character vector indicating name of function of theoretical model to be used. For example, for Generalized_Yule(n, rho, alpha), model_fn_name <- 'Generalized Yule'

iter Integer indicating number of iterations to run grid search. Increasing iterations will increase decimal point precision of output parameters.

weighted_rt Boolean used to determine if the weighted right-tail cumulative distribution function should be used or not.

n_cores Integer used to indicate number of cores to be used for this function if a socket cluster object is not defined.

`clust` socket cluster object from `'parallel::makeCluster()'`. This is used if you have already generated a socket cluster object and would like to run this function on it. If no object is defined, one will be made for this function call.

`global_fit_RGHD_ratio` *Global optimization of the 2m-RGHD function given empirical data, r bounds, and q/r bounds.*

Description

This function generates a single set of optimized parameters and Psi Criterion for a given function within specified starting parameter bounds. This function uses Limited Memory BFGS as its gradient descent algorithm.

Usage

```
global_fit_RGHD_ratio(
  param_bounds,
  data,
  iter,
  weighted_rt = FALSE,
  n_cores = 1,
  clust
)
```

Arguments

<code>param_bounds</code>	A list of sequences which indicate space where parameters should be generated and fit
<code>data</code>	Vector of observed values
<code>iter</code>	Integer indicating number of iterations to run grid search. Increasing iterations will increase decimal point precision of output parameters.
<code>weighted_rt</code>	Boolean used to determine if the weighted right-tail cumulative distribution function should be used or not.
<code>n_cores</code>	Integer used to indicate number of cores to be used for this function if a socket cluster object is not defined.
<code>clust</code>	socket cluster object from <code>'parallel::makeCluster()'</code> . This is used if you have already generated a socket cluster object and would like to run this function on it. If no object is defined, one will be made for this function call.

Kolmogorov_Waring *Kolmogorov Waring*

Description

Calculates vector of n length of Kolmogorov distribution function given parameters

Usage

Kolmogorov_Waring(n, a, b, theta)

Arguments

n	Length of vector to be generated
a	Parameter of the Kolmogorov Waring distribution function
b	Parameter of the Kolmogorov Waring distribution function
theta	Parameter of the Kolmogorov Waring distribution function

Kolmogorov_Waring_P0 *Kolmogorov Waring P0*

Description

Calculates P0 of Kolmogorov Waring distribution function given parameters. Approximation is used if parameters meet a specific criteria.

Usage

Kolmogorov_Waring_P0(a, b, theta)

Arguments

a	Parameter of the Kolmogorov Waring distribution function
b	Parameter of the Kolmogorov Waring distribution function
theta	Parameter of the Kolmogorov Waring distribution function

 Kolmogorov_Waring_P0_calc

Kolmogorov Waring P0 calculation

Description

Calculates P0 of Kolmogorov Waring distribution function given parameters

Usage

```
Kolmogorov_Waring_P0_calc(a, b, theta)
```

Arguments

a	Parameter of the Kolmogorov Waring distribution function
b	Parameter of the Kolmogorov Waring distribution function
theta	Parameter of the Kolmogorov Waring distribution function

local_fit_function	<i>Local optimization of a given function given empirical data and parameter bounds</i>
--------------------	---

Description

This function generates a table of optimized parameters and Psi Criterion for a given function within specified starting parameter bounds. This function uses Limited Memory BFGS as it's gradient descent algorithm.

Usage

```
local_fit_function(
  param_bounds,
  data,
  model_fn_name,
  weighted_rt = FALSE,
  par_chunk = 100,
  par_chunk_size = 10,
  n_cores = 1,
  clust,
  left_trunc = 1,
  right_trunc = left_trunc + length(data) - 1
)
```

Arguments

param_bounds	A list of sequences which indicate space where parameters should be generated and fit
data	Vector of observed values
model_fn_name	Character vector indicating name of function of theoretical model to be used. For example, for Generalized_Yule(n, rho, alpha), model_fn_name <- 'Generalized Yule'
weighted_rt	Boolean used to determine if the weighted right-tail cumulative distribution function should be used or not.
par_chunk	Integer used to indicate number of optimization chunks to be run. Total number of rows in the output table = par_chunk * par_chunk_size
par_chunk_size	Integer used to indicate number of starting parameters to be generated and optimized in a given chunk. Total number of rows in the output table = par_chunk * par_chunk_size
n_cores	Integer used to indicate number of cores to be used for this function if a socket cluster object is not defined.
clust	socket cluster object from 'parallel::makeCluster()'. This is used if you have already generated a socket cluster object and would like to run this function on it. If no object is defined, one will be made for this function call.
left_trunc	Int used to determine starting index of model to use for optimization
right_trunc	Int used to determine ending index of model to use for optimization

local_fit_RGHD_ratio *Local optimization of the 2m-RGHD function given empirical data, r bounds, and q/r bounds.*

Description

This function generates a table of optimized parameters and Psi Criterion for a given function within specified starting parameter bounds. This function uses Limited Memory BFGS as it's gradient descent algorithm.

Usage

```
local_fit_RGHD_ratio(
  param_bounds,
  data,
  weighted_rt = FALSE,
  par_chunk = 100,
  par_chunk_size = 10,
  n_cores = 1,
  clust,
  left_trunc = 1,
  right_trunc = left_trunc + length(data) - 1
)
```

Arguments

param_bounds	A list of sequences which indicate space where parameters should be generated and fit
data	Vector of observed values
weighted_rt	Boolean used to determine if the weighted right-tail cumulative distribution function should be used or not.
par_chunk	Integer used to indicate number of optimization chunks to be run. Total number of rows in the output table = par_chunk * par_chunk_size
par_chunk_size	Integer used to indicate number of starting parameters to be generated and optimized in a given chunk. Total number of rows in the output table = par_chunk * par_chunk_size
n_cores	Integer used to indicate number of cores to be used for this function if a socket cluster object is not defined.
clust	socket cluster object from 'parallel::makeCluster()'. This is used if you have already generated a socket cluster object and would like to run this function on it. If no object is defined, one will be made for this function call.
left_trunc	Int used to determine starting index of model to use for optimization
right_trunc	Int used to determine ending index of model to use for optimization

Lorentzian

Lorentzian Distribution Function

Description

This function generates a vector of n length of the Lorentzian distribution

Usage

```
Lorentzian(n, gamma, x0, c)
```

Arguments

n	Length of vector to be generated.
gamma	Parameter of the Lorentzian distribution function
x0	Parameter of the Lorentzian distribution function indicating center of function
c	Parameter of the Lorentzian distribution function indicating center of function

Examples

```
Lorentzian_calc(5, 5.5, 6, 2)
```

Lorentzian_calc	<i>Lorentzian Distribution Function calculation</i>
-----------------	---

Description

This function calculates value of Lorentzian function at x

Usage

```
Lorentzian_calc(x, gamma, x0, c)
```

Arguments

x	Index of function
gamma	Parameter of the Lorentzian distribution function
x0	Parameter of the Lorentzian distribution function indicating center of function
c	Parameter of the Lorentzian distribution function indicating center of function

Examples

```
Lorentzian_calc(5, 5.5, 6, 2)
```

parameter_post_processing	<i>Parameter Optimization Helper Function</i>
---------------------------	---

Description

This function adds in additional columns to the optimized parameter output dataframe

Usage

```
parameter_post_processing(parameter_df, model_fn_name, data)
```

Arguments

parameter_df	Output dataframe of optimized parameters using local algorithm
model_fn_name	Character vector used to indicate name of model function used for optimization
data	Vector of observed values

plot_model	<i>Plot Model Helper Function</i>
------------	-----------------------------------

Description

This function generates various plots of empirical data and models

Usage

```
plot_model(  
  title,  
  model_fn_name,  
  data,  
  parameter_df,  
  n_parameters,  
  plot_folder_name,  
  xlab,  
  left_trunc = 1  
)
```

Arguments

title	Character vector indicating title of the empirical dataset, this will be present on every plot, this also determines the name of the folder where plots will be
model_fn_name	Character vector used to indicate name of model function used for optimization
data	Vector of observed values
parameter_df	Data frame of optimized parameters and other model function values (p0, Psi, etc)
n_parameters	Int of number of parameters used in model function
plot_folder_name	Character vector indicating folder or directory name to be used when outputting plot images
xlab	Character vector indicating x axis label of plots, indicates what the random variable is
left_trunc	Int indicating starting index of model function used for optimization

psi_criterion	<i>Psi Criterion</i>
---------------	----------------------

Description

This function generates the Psi Criterion goodness of fit value given an empirical distribution, theoretical modeled distribution, and number of parameters in the theoretical distribution.

Usage

```
psi_criterion(data, model, n_parameters)
```

Arguments

data	Vector of observed values
model	Vector of theoretical values to be compared
n_parameters	Number of parameters of function used to generate model

Examples

```
obs_data <- c(100,75,20,1)
model_data <- Kolmogorov_Waring(length(obs_data), 2, 3, 0.9)
psi <- psi_criterion(obs_data, model_data, 3)
```

psi_criterion_function	<i>Psi Criterion given a function</i>
------------------------	---------------------------------------

Description

This function generates the Psi Criterion goodness of fit value given an empirical distribution. The function and parameters are given, as well as desired weight of pmf and use of the weighted right-tail cumulative distribution function.

Usage

```
psi_criterion_function(
  params,
  data,
  model_fn,
  pmf_weight = 0,
  weighted_rt = FALSE,
  left_trunc = 1,
  right_trunc = left_trunc + length(data) - 1
)
```

Arguments

params	Vector of parameters for model_fn, not including n. For example, for Generalized_Yule(n, rho, alpha), params will be c(rho, alpha)
data	Vector of observed values
model_fn	Function of theoretical model to be used. For example, for Generalized_Yule(n, rho, alpha), model_fn <- Generalied_Yule
pmf_weight	Numeric of weight given to probability mass function for generation of Psi Criterion. For example, if pmf_weight <- 0.5, 50 percent of the Psi Criterion value will be attributed to the probability mass function while the other 50 percent will be attributed to the right-tail cumulative distribution function.
weighted_rt	Boolean used to determine if the weighted right-tail cumulative distribution function should be used or not.
left_trunc	Int used to determine starting index of model to use for optimization
right_trunc	Int used to determine ending index of model to use for optimization

Examples

```
obs_data <- c(100,75,20,1)
parameters <- c(1,2,0.8)
psi <- psi_criterion_function(parameters, obs_data, Kolmogorov_Waring)
```

psi_criterion_RGHD_ratio

Psi Criterion for RGHD parameter ratios

Description

This function generates the Psi Criterion goodness of fit value given an empirical distribution for the 2m-RGHD function. Parameters r and q/r ratios are given, as well as desired weight of pmf and use of the weighted right-tail cumulative distribution function.

Usage

```
psi_criterion_RGHD_ratio(
  params,
  data,
  m,
  pmf_weight = 0,
  weighted_rt = FALSE,
  left_trunc = 1,
  right_trunc = left_trunc + length(data) - 1
)
```


Arguments

params	Vector of parameters for model_fn, not including n. For example, for 2m-RGHD (m=2), params <- c(3, 5, 0.3, 1.5). In this case r1 = 3, r2 = 5, q1/r1 = 0.3, and q2/r2 = 1.5
data	Vector of observed values
m	m parameter for 2m-RGHD function
pmf_weight	Numeric of weight given to probability mass function for generation of Psi Criterion. For example, if pmf_weight <- 0.5, 50 percent of the Psi Criterion value will be attributed to the probability mass function while the other 50 percent will be attributed to the right-tail cumulative distribution function.
weighted_rt	Boolean used to determine if the weighted right-tail cumulative distribution function should be used or not.
left_trunc	Int used to determine starting index of model to use for optimization
right_trunc	Int used to determine ending index of model to use for optimization

Examples

```
obs_data <- c(100,75,20,1)
parameters <- c(3, 5, 0.3, 1.5)
psi <- psi_criterion_RGHD_ratio(parameters, obs_data, 2)
```

RGHD

2m-RGHD Distribution Function

Description

Returns doubly truncated vector of 2m-RGHD function values where input is 1-J

Usage

```
RGHD(J, m, r, q, P0_iter = 100L, P0_included = FALSE)
```

Arguments

J	Length of vector to be generated
m	Parameter of the 2m-RGHD function, this defines number of r and q parameters of the function
r	R vector containing r parameters from 1:m
q	R vector containing q parameters from 1:m
P0_iter	Integer indicating number of iterations to use for calculation of P0, increasing this parameter will increase accuracy of P0
P0_included	Boolean used to include P0 in vector or not

 RGHD_P0

2m-RGHD Distribution Function P0

Description

Calculates P0 given a set of parameters

Usage

RGHD_P0(m, r, q)

Arguments

m	Parameter of the 2m-RGHD function, this defines number of r and q parameters of the function
r	R vector containing r parameters from 1:m
q	R vector containing q parameters from 1:m

 RGHD_P0_calc

2m-RGHD Distribution Function P0 calculation

Description

Calculates P0 given a set of parameters

Usage

RGHD_P0_calc(sigma_upper, m, r, q)

Arguments

sigma_upper	Int which determine number of iterations for calculation to go through, this is needed to approximate sigma infinity
m	Parameter of the 2m-RGHD function, this defines number of r and q parameters of the function
r	R vector containing r parameters from 1:m
q	R vector containing q parameters from 1:m

right_tail_cdf	<i>Right-Tail Cumulative Distribution Function</i>
----------------	--

Description

This function generates a vector of the right-tail cumulative distribution function of a given vector of values.

Usage

```
right_tail_cdf(x)
```

Arguments

x Length of vector to be generated.

Examples

```
x <- c(1,2,3,4,5)
right_tail_cdf(x)
```

skeweDF_auto	<i>SkeweDF Auto Helper Function</i>
--------------	-------------------------------------

Description

This function will automatically optimize parameters for an empirical dataset given a model function and generate plots and tables

Usage

```
skeweDF_auto(
  title = "Dataset",
  data,
  xlab = "Random Variable",
  param_bounds,
  model_fn_name,
  left_trunc = 1,
  right_trunc = left_trunc + length(data) - 1,
  n_cores = 1
)
```

Arguments

title	Character vector indicating title of the empirical dataset, this will be present on every plot, this also determines the name of the folder where plots will be
data	Vector of observed values
xlab	Character vector indicating x axis label of plots, indicates what the random variable is
param_bounds	A list of sequences which indicate space where parameters should be generated and fit
model_fn_name	Character vector used to indicate name of model function used for optimization
left_trunc	Int used to determine starting index of model to use for optimization
right_trunc	Int used to determine ending index of model to use for optimization
n_cores	Integer used to indicate number of cores to be used for this function if a socket cluster object is not defined.

`weighted_left_tail_cdf`*Weighted Left-Tail Cumulative Distribution Function*

Description

This function generates a vector of the weighted left-tail cumulative distribution function of a given vector of values. The weight of each variable is determined by its position in the vector. For example, with a vector of length 5, element 1 will have weight $5/(5+4+3+2+1)$. Element 1 will have weight $5/(5+4+3+2+1)$

Usage

```
weighted_left_tail_cdf(x)
```

Arguments

x	Length of vector to be generated.
---	-----------------------------------

Examples

```
x <- c(1,2,3,4,5)
weighted_left_tail_cdf(x)
```

`weighted_right_tail_cdf`*Weighted Right-Tail Cumulative Distribution Function*

Description

This function generates a vector of the weighted right-tail cumulative distribution function of a given vector of values. The weight of each variable is determined by its position in the vector. For example, with a vector of length 5, element 5 will have weight $5/(5+4+3+2+1)$. Element 1 will have weight $1/(5+4+3+2+1)$.

Usage

```
weighted_right_tail_cdf(x)
```

Arguments

`x` Length of vector to be generated.

Examples

```
x <- c(1,2,3,4,5)
weighted_right_tail_cdf(x)
```

`write_input_table`*Write Input Table Helper Function*

Description

This function generates table of input data

Usage

```
write_input_table(folder_name, data)
```

Arguments

`folder_name` Character vector indicating folder or directory name to be used when outputting table

`data` Vector of observed values

 write_parameter_table *Write Parameter Table Helper Function*

Description

This function generates table of optimized parameters

Usage

```
write_parameter_table(parameter_df, folder_name, model_fn_name, RGHD_m = 0)
```

Arguments

parameter_df	Data frame of optimized parameters and other model function values (p0, Psi, etc)
folder_name	Character vector indicating folder or directory name to be used when outputting table
model_fn_name	Character vector used to indicate name of model function used for optimization
RGHD_m	Int indicating m value of 2m-RGHD function if applicable

 write_summary_table *Write Summary Table Helper Function*

Description

This function generates summary statistics table of optimized parameters

Usage

```
write_summary_table(parameter_df, folder_name, model_fn_name, RGHD_m = 0)
```

Arguments

parameter_df	Data frame of optimized parameters and other model function values (p0, Psi, etc)
folder_name	Character vector indicating folder or directory name to be used when outputting table
model_fn_name	Character vector used to indicate name of model function used for optimization
RGHD_m	Int indicating m value of 2m-RGHD function if applicable

Yule

Yule Distribution Function

Description

This function generates a vector of n length of the Yule distribution with parameter rho.

Usage

```
Yule(n, rho)
```

Arguments

n	Length of vector to be generated.
rho	Parameter of the Yule distribution function

Examples

```
Yule(100, 3)
```

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