

The General Distance Measure (see Walesiak [2006])

The General Distance Measure (*GDM*) is given by the following equation:

$$GDM = \frac{1}{2} - \frac{\sum_{j=1}^m a_{ikj} b_{kij} + \sum_{j=1}^m \sum_{\substack{l=1 \\ l \neq i, k}}^n a_{ilj} b_{klj}}{2 \left[\sum_{j=1}^m \sum_{l=1}^n a_{ilj}^2 \cdot \sum_{j=1}^m \sum_{l=1}^n b_{klj}^2 \right]^{\frac{1}{2}}}, \quad (1)$$

where: $i, k, l = 1, \dots, n$ – the number of object,

$j = 1, \dots, m$ – the number of variable,

$x_{ij} (x_{kj}, x_{lj})$ – i -th (k -th, l -th) observation on j -th variable.

For the variables measured on **ratio and (or) interval scale** we take a_{ipj} , b_{krj} given as (Walesiak [2002]):

$$\begin{aligned} a_{ipj} &= x_{ij} - x_{pj} & \text{for } p &= k, l \\ b_{krj} &= x_{kj} - x_{rj} & \text{for } r &= i, l \end{aligned} \quad (2)$$

$$GDM1 = \frac{1}{2} - \frac{\sum_{j=1}^m (x_{ij} - x_{kj})(x_{kj} - x_{ij}) + \sum_{j=1}^m \sum_{\substack{l=1 \\ l \neq i, k}}^n (x_{ij} - x_{lj})(x_{kj} - x_{lj})}{2 \left[\sum_{j=1}^m \sum_{l=1}^n (x_{ij} - x_{lj})^2 \cdot \sum_{j=1}^m \sum_{l=1}^n (x_{kj} - x_{lj})^2 \right]^{\frac{1}{2}}}. \quad (3)$$

For the variables measured on **ordinal scale** we take a_{ipj} , b_{krj} given as (Walesiak [1993], pp. 44-45):

$$a_{ipj} (b_{krj}) = \begin{cases} 1 & \text{if } x_{ij} > x_{pj} \left(x_{kj} > x_{rj} \right) \\ 0 & \text{if } x_{ij} = x_{pj} \left(x_{kj} = x_{rj} \right), \\ -1 & \text{if } x_{ij} < x_{pj} \left(x_{kj} < x_{rj} \right) \end{cases} \quad \text{for } p = k, l; l, r = i, l, \quad (4)$$

$$GDM2 = \frac{1}{2} - \frac{\sum_{j=1}^m a_{ikj} b_{kij} + \sum_{j=1}^m \sum_{\substack{l=1 \\ l \neq i, k}}^n a_{ilj} b_{klj}}{2 \left[\sum_{j=1}^m \sum_{l=1}^n a_{ilj}^2 \cdot \sum_{j=1}^m \sum_{l=1}^n b_{klj}^2 \right]^{\frac{1}{2}}}. \quad (5)$$

The properties of the General Distance Measure

- it can be applied when the variables are measured on the ordinal (*GDM2*), interval and ratio scale (*GDM1*),
- it takes values from the $[0; 1]$ interval. Value 0 indicates that for the compared objects i, k between corresponding observations of variables, only relations “equal to” take place. For *GDM2* the value 1 indicates that for the compared objects i, k between corresponding observations on ordinal variables, relations “greater than” take place or relations “greater than” and relations “equal to”, if they are held for other objects (i.e. objects numbered $l = 1, \dots, n$; where $l \neq i, k$),

- it satisfies the conditions: non-negative, reflexive, and symmetric (for all $i, k = 1, \dots, n$),
- the empirical analysis proves that distance GDM sometimes does not satisfy the triangle inequality,
- it needs at least one pair of non-identical objects in order to avoid zero in the denominator,
- the transformation of data by any strictly increasing function ($GDM2$) or by any linear function ($GDM1$) does not change the value of distance.

References

- Jajuga, K., Walesiak, M., Bak, A. (2003), *On the general distance measure*, In: M. Schwaiger, O. Opitz (Eds.), *Exploratory data analysis in empirical research*, Springer-Verlag, Berlin, Heidelberg, 104-109.
- Walesiak, M. (1993), *Statystyczna analiza wielowymiarowa w badaniach marketingowych [Multivariate Statistical Analysis in Marketing Research]*, Wrocław University of Economics, Research Papers no. 654.
- Walesiak, M. (2002), *Propozycja uogólnionej miary odległości w statystycznej analizie wielowymiarowej*, In: J. Paradysz (Ed.), *Statystyka regionalna w służbie samorządu lokalnego i biznesu*, Internetowa Oficyna Wydawnicza, Centrum Statystyki Regionalnej, AE w Poznaniu, Poznań, 115-121.
- Walesiak, M. (2006), *Uogólniona miara odległości w statystycznej analizie wielowymiarowej [The Generalized Distance Measure in multivariate statistical analysis]*, Wydanie drugie rozszerzone, Wydawnictwo AE, Wrocław.