

# Examples from Multilevel Software Comparative Reviews

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## Abstract

The Center for Multilevel Modelling at the Institute of Education, London maintains a web site of “Software reviews of multilevel modeling packages”. The data sets discussed in the reviews are available at this web site. We have incorporated these data sets in the `lme4` package for R and, in this vignette, provide the results of fitting several models to these data sets.

## 1 Introduction

## 2 Two-level normal models

The Exam data set is used in fitting examples of two-level normal multilevel models.

```
> data(Exam)
> str(Exam)
`data.frame`:      4059 obs. of  9 variables:
 $ school  : Factor w/ 65 levels "1","2","3","4",...: 1 1 1 1 1 1 1 1 1 1 ...
 $ student : Factor w/ 650 levels "1","2","3","4",...: 143 145 142 141 138 155 158 115 117 113 ...
 $ normexam: num   0.261  0.134 -1.724  0.968  0.544 ...
 $ standlrt: num   0.619  0.206 -1.365  0.206  0.371 ...
 $ gender  : Factor w/ 2 levels "F","M": 1 1 2 1 1 2 2 2 1 2 ...
 $ schgend : Factor w/ 3 levels "mixed","boys",...: 1 1 1 1 1 1 1 1 1 1 ...
 $ schavg  : num   0.166  0.166  0.166  0.166  0.166 ...
 $ vr      : Factor w/ 3 levels "bottom 25%","mi...",...: 2 2 2 2 2 2 2 2 2 2 ...
 $ intake  : Factor w/ 3 levels "bottom 25%","mi...",...: 1 2 3 2 2 1 3 2 2 3 ...
```

```

> system.time(mE1 <- lme(normexam ~ standlrt + gender + schgend,
+   Exam, ~1 | school), gc = TRUE)
[1] 0.08 0.01 0.09 0.00 0.00
> summary(mE1)
Linear mixed-effects model fit by REML
Fixed: normexam ~ standlrt + gender + schgend
Data: Exam
      AIC      BIC    logLik
9361.673 9405.834 -4673.837

Random effects:
Groups   Name             Variance Std.Dev.
school   (Intercept) 0.085829 0.29297
Residual                      0.56253 0.75002

Fixed effects:
              Estimate Std. Error  DF t value Pr(>|t|)
(Intercept) -1.0493e-03  5.5569e-02 4054 -0.0189  0.98494
standlrt     5.5975e-01  1.2450e-02 4054 44.9601 < 2.2e-16
genderM      -1.6739e-01  3.4100e-02 4054 -4.9089 9.519e-07
schgendboys  1.7769e-01  1.1347e-01 4054  1.5659  0.11745
schgendgirls 1.5900e-01  8.9403e-02 4054  1.7784  0.07541

Correlation of Fixed Effects:
              (Intr) stndlr gendrM schgndb
standlrt     -0.014
genderM       -0.316  0.061
schgendboys  -0.395 -0.003 -0.145
schgendgirls -0.622  0.009  0.197  0.245

Number of Observations: 4059
Number of Groups: 65

> system.time(mE2 <- lme(normexam ~ standlrt * gender + schgend,
+   Exam, ~1 | school), gc = TRUE)
[1] 0.05 0.00 0.05 0.00 0.00
> summary(mE2)
Linear mixed-effects model fit by REML
Fixed: normexam ~ standlrt * gender + schgend
Data: Exam
      AIC      BIC    logLik
9369.204 9419.673 -4676.602

Random effects:
Groups   Name             Variance Std.Dev.
school   (Intercept) 0.085856 0.29301
Residual                      0.56267 0.75011

Fixed effects:
              Estimate Std. Error  DF t value Pr(>|t|)
(Intercept)  -8.4349e-04  5.5586e-02 4053 -0.0152  0.98789
standlrt      5.5745e-01  1.6662e-02 4053 33.4572 < 2.2e-16
genderM       -1.6733e-01  3.4105e-02 4053 -4.9064 9.638e-07
schgendboys   1.7765e-01  1.1349e-01 4053  1.5653  0.11759
schgendgirls  1.5879e-01  8.9422e-02 4053  1.7757  0.07586
standlrt:genderM 5.1121e-03  2.4584e-02 4053  0.2079  0.83528

```

```

Correlation of Fixed Effects:
              (Intr) stndlr gendrM schgnb schgndg
standlrt      -0.022
genderM       -0.316  0.040
schgendboys  -0.395 -0.001 -0.145
schgendgrls  -0.622  0.014  0.196  0.245
stndlrt:gnM   0.018 -0.664  0.008 -0.002 -0.011

Number of Observations: 4059
Number of Groups: 65

> system.time(mE3 <- lme(normexam ~ standlrt * gender + schgend,
+   Exam, ~standlrt | school), gc = TRUE)
[1] 0.06 0.00 0.06 0.00 0.00

> summary(mE3)

Linear mixed-effects model fit by REML
Fixed: normexam ~ standlrt * gender + schgend
Data: Exam
      AIC      BIC    logLik
9328.242 9391.329 -4654.121

Random effects:
Groups Name      Variance Std.Dev. Corr
school (Intercept) 0.083723 0.28935
      standlrt    0.015250 0.12349  0.575
Residual              0.55037  0.74187

Fixed effects:
              Estimate Std. Error  DF t value  Pr(>|t|)
(Intercept)  -2.1277e-02  5.3279e-02 4053 -0.3993   0.68966
standlrt      5.5713e-01  2.4349e-02 4053 22.8812 < 2.2e-16
genderM      -1.6859e-01  3.3844e-02 4053 -4.9814 6.576e-07
schgendboys  1.7751e-01  1.0211e-01 4053  1.7384  0.08221
schgendgirls 1.7790e-01  8.2104e-02 4053  2.1668  0.03031
standlrt:genderM -6.8757e-03  2.9540e-02 4053 -0.2328  0.81596

Correlation of Fixed Effects:
              (Intr) stndlr gendrM schgnb schgndg
standlrt      0.200
genderM       -0.337  0.026
schgendboys  -0.354 -0.048 -0.148
schgendgrls  -0.600  0.116  0.225  0.218
stndlrt:gnM   0.067 -0.559  0.010  0.094 -0.181

Number of Observations: 4059
Number of Groups: 65

```

There are some interesting aspects of data management that show up in the analysis of these data. The `student` variable is an identifier of the student within the `school`. It would be best to combine the indicators of school and student to get a unique identifier of the student.

```

> Exam$ids <- (Exam$school:Exam$student)[, drop = TRUE]
> str(Exam)

```

```

`data.frame':      4059 obs. of  10 variables:
 $ school  : Factor w/ 65 levels "1","2","3","4",...: 1 1 1 1 1 1 1 1 1 1 ...
 $ student : Factor w/ 650 levels "1","2","3","4",...: 143 145 142 141 138 155 158 115 117 113 ...
 $ normexam: num   0.261  0.134 -1.724  0.968  0.544 ...
 $ standlrt: num   0.619  0.206 -1.365  0.206  0.371 ...
 $ gender  : Factor w/ 2 levels "F","M": 1 1 2 1 1 2 2 2 1 2 ...
 $ schgend : Factor w/ 3 levels "mixed","boys",...: 1 1 1 1 1 1 1 1 1 1 ...
 $ schavg  : num   0.166 0.166 0.166 0.166 0.166 ...
 $ vr      : Factor w/ 3 levels "bottom 25%","mi..",...: 2 2 2 2 2 2 2 2 2 2 ...
 $ intake  : Factor w/ 3 levels "bottom 25%","mi..",...: 1 2 3 2 2 1 3 2 2 3 ...
 $ ids     : Factor w/ 4055 levels "1:1","1:4","1:6",...: 48 49 47 46 45 50 51 39 40 38 ...

```

Notice that there are 4059 observations but only 4055 unique levels of student within school. We can check the ones that are duplicated

```

> Exam$ids[which(duplicated(Exam$ids))]
[1] 43:86 50:39 52:2  52:21
4055 Levels: 1:1 1:4 1:6 1:7 1:13 1:14 1:16 1:17 1:19 1:22 1:27 ... 65:155

```

One of these duplicated cases is particularly interesting. One of the students with the duplicated student id 86 in school 43 is the only male student in this mixed school. This is probably a case of a misrecorded school.

### 3 Three-level Normal Models

Data from the 1997 A-level Chemistry exam are available as Chem97.

```

> data(Chem97)
> str(Chem97)
`data.frame':      31022 obs. of  8 variables:
 $ lea      : Factor w/ 131 levels "1","2","3","4",...: 1 1 1 1 1 1 1 1 1 1 ...
 $ school   : Factor w/ 2410 levels "1","2","3","4",...: 1 1 1 1 1 1 1 1 1 1 ...
 $ student  : Factor w/ 31022 levels "1","2","3","4",...: 1 2 3 4 5 6 7 8 9 10 ...
 $ score    : num   4 10 10 10 8 10 6 8 4 10 ...
 $ gender   : Factor w/ 2 levels "M","F": 2 2 2 2 2 2 2 2 2 2 ...
 $ age      : num   3 -3 -4 -2 -1 4 1 4 3 0 ...
 $ gcscscore: num   6.62 7.62 7.25 7.50 6.44 ...
 $ gcsecnt  : num   0.339 1.339 0.964 1.214 0.158 ...

> system.time(mC1 <- lme(score ~ 1, Chem97, ~1 | lea/school))
[1] 0.84 0.05 0.89 0.00 0.00

> summary(mC1)
Linear mixed-effects model fit by REML
Fixed: score ~ 1
Data: Chem97
      AIC      BIC    logLik
157881.8 157915.2 -78936.9

Random effects:
Groups   Name      Variance Std.Dev.
school   (Intercept) 2.74872  1.65793
lea      (Intercept) 0.15349  0.39178

```

```

Residual                8.5161    2.9182

Fixed effects:
      Estimate Std. Error    DF t value  Pr(>|t|)
(Intercept) 5.3190e+00 5.8108e-02 31021  91.536 < 2.2e-16

Number of Observations: 31022
Number of Groups:
school    lea
  2410    131

> system.time(mC2 <- lme(score ~ gcsecnt, Chem97, ~1 | lea/school))
[1] 0.92 0.03 0.95 0.00 0.00
> summary(mC2)

Linear mixed-effects model fit by REML
Fixed: score ~ gcsecnt
Data: Chem97
      AIC      BIC    logLik
141707 141748.7 -70848.5

Random effects:
Groups   Name      Variance Std.Dev.
school  (Intercept) 1.166198 1.07991
lea     (Intercept) 0.014766 0.12151
Residual                    5.1542  2.2703

Fixed effects:
      Estimate Std. Error    DF t value  Pr(>|t|)
(Intercept) 5.6355e+00 3.1235e-02 31020  180.42 < 2.2e-16
gcsecnt     2.4726e+00 1.6904e-02 31020  146.27 < 2.2e-16

Correlation of Fixed Effects:
      (Intr)
gcsecnt 0.058

Number of Observations: 31022
Number of Groups:
school    lea
  2410    131

```

## 4 Two-level models for binary data

The data frame `Contraception` provides data from the Bangladesh fertility survey.

```

> data(Contraception)
> str(Contraception)

`data.frame`:      1934 obs. of  6 variables:
 $ woman   : Factor w/ 1934 levels "1","2","3","4",...: 1 2 3 4 5 6 7 8 9 10 ...
 $ district: Factor w/ 60 levels "1","2","3","4",...: 1 1 1 1 1 1 1 1 1 1 ...
 $ use      : Factor w/ 2 levels "N","Y": 1 1 1 1 1 1 1 1 1 1 ...
 $ livch    : Factor w/ 4 levels "0","1","2","3+": 4 1 3 4 1 1 4 4 2 4 ...
 $ age      : num  18.44 -5.56 1.44 8.44 -13.56 ...
 $ urban     : Factor w/ 2 levels "N","Y": 2 2 2 2 2 2 2 2 2 2 ...

```

```

> summary(Contraception[, -1])

    district    use    livch    age    urban
14      : 118  N:1175    0 :530  Min.   :-13.560000  N:1372
1       : 117  Y: 759    1 :356  1st Qu.: -7.559900  Y: 562
46      :  86                2 :305  Median : -1.559900
25      :  67                3+:743  Mean   :  0.002198
6       :  65                3rd Qu.:  6.440000
30      :  61                Max.   : 19.440000
(Other):1420

> system.time(mB1 <- GLMM(use ~ urban + age + livch, binomial,
+   Contraception, ~1 | district))

Iteration 1 Termination Criterion: 0.3280704
Iteration 2 Termination Criterion: 0.04547198
Iteration 3 Termination Criterion: 0.0009326131
Iteration 4 Termination Criterion: 3.003239e-06
Iteration 5 Termination Criterion: 1.591316e-08
Iteration 6 Termination Criterion: 7.715577e-09
[1] 0.55 0.00 0.56 0.00 0.00

> summary(mB1)

Generalized Linear Mixed Model

Family: binomial family with logit link
Fixed: use ~ urban + age + livch
Data: Contraception
      AIC      BIC    logLik
2429.664 2474.203 -1206.832

Random effects:
Groups   Name      Variance Std.Dev.
district (Intercept) 0.21518  0.46387

Estimated scale (compare to 1)  0.9844111

Fixed effects:
              Estimate Std. Error z value Pr(>|z|)
(Intercept) -1.6606460  0.1452147 -11.4358 < 2.2e-16
urbanY       0.7193097  0.1183317  6.0788 1.211e-09
age         -0.0261558  0.0078152 -3.3468 0.0008176
livch1       1.0921026  0.1565011  6.9782 2.989e-12
livch2       1.3545533  0.1729641  7.8314 4.824e-15
livch3+      1.3241531  0.1773558  7.4661 8.262e-14

Correlation of Fixed Effects:
      (Intr) urbanY age    livch1 livch2
urbanY -0.300
age      0.446 -0.046
livch1  -0.589  0.059 -0.211
livch2  -0.631  0.094 -0.378  0.488
livch3+ -0.748  0.098 -0.674  0.539  0.619

Number of Observations: 1934
Number of Groups: 60

> system.time(mB2 <- GLMM(use ~ urban + age + livch, binomial,
+   Contraception, ~1 | district, method = "Laplace"))

```

```

Iteration 1 Termination Criterion: 0.3280704
Iteration 2 Termination Criterion: 0.04547198
Iteration 3 Termination Criterion: 0.0009326131
Iteration 4 Termination Criterion: 3.003239e-06
Iteration 5 Termination Criterion: 1.591316e-08
Iteration 6 Termination Criterion: 7.715577e-09
Using optimizer nlm
[1] 31.29 0.05 31.34 0.00 0.00
> summary(mB2)
Generalized Linear Mixed Model

Family: binomial family with logit link
Fixed: use ~ urban + age + livch
Data: Contraception
      AIC      BIC    logLik
2417.616 2428.750 -1206.808

Random effects:
Groups   Name      Variance Std.Dev.
district (Intercept) 0.21239  0.46086

Estimated scale (compare to 1) 0.9859618

Fixed effects:
              Estimate Std. Error z value Pr(>|z|)
(Intercept) -1.6897106  0.1459307 -11.5789 < 2.2e-16
urbanY       0.7329915  0.1192198  6.1482 7.835e-10
age         -0.0265950  0.0078773  -3.3762 0.000735
livchl      1.1091842  0.1576922  7.0339 2.009e-12
livch2      1.3763954  0.1743349  7.8951 2.900e-15
livch3+     1.3452344  0.1787129  7.5274 5.178e-14

Correlation of Fixed Effects:
      (Intr) urbanY age    livchl livch2
urbanY -0.301
age      0.448 -0.046
livchl  -0.589  0.059 -0.210
livch2  -0.631  0.094 -0.378  0.487
livch3+ -0.749  0.099 -0.674  0.538  0.618

Number of Observations: 1934
Number of Groups: 60
> system.time(mB3 <- GLMM(use ~ urban + age + livch, family = binomial,
+   data = Contraception, random = ~urban / district))
Iteration 1 Termination Criterion: 0.4616156
Iteration 2 Termination Criterion: 0.07803401
Iteration 3 Termination Criterion: 0.00324362
Iteration 4 Termination Criterion: 7.447095e-05
Iteration 5 Termination Criterion: 1.521883e-06
Iteration 6 Termination Criterion: 1.870136e-07
Iteration 7 Termination Criterion: 8.472963e-08
Iteration 8 Termination Criterion: 4.508101e-08
Iteration 9 Termination Criterion: 3.593155e-08
Iteration 10 Termination Criterion: 2.878334e-08
Iteration 11 Termination Criterion: 2.312839e-08
Iteration 12 Termination Criterion: 1.862046e-08

```

```

Iteration 13 Termination Criterion: 1.501004e-08
Iteration 14 Termination Criterion: 1.211791e-08
[1] 0.72 0.00 0.72 0.00 0.00
> summary(mB3)

Generalized Linear Mixed Model

Family: binomial family with logit link
Fixed: use ~ urban + age + livch
Data: Contraception
      AIC      BIC    logLik
2225.720 2281.394 -1102.860

Random effects:
Groups   Name      Variance Std.Dev. Corr
district (Intercept) 0.38774  0.62269
          urbanY      0.66745  0.81698 -0.793

Estimated scale (compare to 1) 0.9759564

Fixed effects:
              Estimate Std. Error z value Pr(>|z|)
(Intercept) -1.6665200  0.1572532 -10.5977 < 2.2e-16
urbanY        0.7914232  0.1681257  4.7073 2.510e-06
age          -0.0258502  0.0079082 -3.2688 0.00108
livch1        1.0987723  0.1580051  6.9540 3.550e-12
livch2        1.3342511  0.1745854  7.6424 2.132e-14
livch3+       1.3227367  0.1795440  7.3672 1.743e-13

Correlation of Fixed Effects:
      (Intr) urbanY age    livch1 livch2
urbanY -0.481
age      0.416 -0.036
livch1  -0.548  0.038 -0.211
livch2  -0.586  0.068 -0.378  0.487
livch3+ -0.695  0.062 -0.674  0.537  0.616

Number of Observations: 1934
Number of Groups: 60

```

## 5 Growth curve model for repeated measures data

```

> data(Oxboys)
> str(Oxboys)

Formal class 'groupedData' [package "lme4"] with 6 slots
 ..@ data :`data.frame`:      234 obs. of  4 variables:
 .. ..$ Subject : Ord.factor w/ 26 levels "10"<"26"<"25"<...: 13 13 13 13 13 13 13 13 13 13 5 ...
 .. ..$ age      : num [1:234] -1.0000 -0.7479 -0.4630 -0.1643 -0.0027 ...
 .. ..$ height   : num [1:234] 140 143 145 147 148 ...
 .. ..$ Occasion: Ord.factor w/ 9 levels "1"<"2"<"3"<"4"<...: 1 2 3 4 5 6 7 8 9 1 ...
 .. ..- attr(*, "FUN")=function(x)
 .. .. ..- attr(*, "source")= chr "function(x) max(x, na.rm = TRUE)"
 ..@ formula:Class 'formula' length 3 height ~ age | Subject

```



```

.. .. attr(*, ".Environment")=length 28 <environment>
..@ outer :Class 'formula' length 2 ~0
.. .. attr(*, ".Environment")=length 3 <environment>
..@ inner :Class 'formula' length 2 ~0
.. .. attr(*, ".Environment")=length 3 <environment>
..@ labels :List of 2
.. ..$ y: chr "Height"
.. ..$ x: chr "Centered age"
..@ units :List of 1
.. ..$ y: chr "(cm)"

> system.time(mX1 <- lme(height ~ age + I(age^2) + I(age^3) +
+ I(age^4), Oxboys, ~age + I(age^2) | Subject), gc = TRUE)
[1] 0.19 0.00 0.19 0.00 0.00

> summary(mX1)

Linear mixed-effects model fit by REML
Fixed: height ~ age + I(age^2) + I(age^3) + I(age^4)
Data: Oxboys
      AIC      BIC    logLik
651.9081 693.372 -313.9541

Random effects:
Groups Name Variance Std.Dev. Corr
Subject (Intercept) 64.03464 8.00216
      age      2.86418 1.69239 0.614
      I(age^2) 0.67429 0.82115 0.215 0.658
Residual      0.21737 0.46623

Fixed effects:
      Estimate Std. Error DF t value Pr(>|t|)
(Intercept) 149.01887 1.57036 229 94.8946 < 2.2e-16
age          6.17418 0.35650 229 17.3187 < 2.2e-16
I(age^2)     1.12823 0.35144 229 3.2103 0.001516
I(age^3)     0.45385 0.16246 229 2.7937 0.005653
I(age^4)     -0.37690 0.30018 229 -1.2556 0.210552

Correlation of Fixed Effects:
      (Intr) age      I(g^2) I(g^3)
age      0.572
I(age^2) 0.076 0.264
I(age^3) -0.001 -0.340 0.025
I(age^4) 0.021 0.016 -0.857 -0.021

Number of Observations: 234
Number of Groups: 26

> system.time(mX2 <- lme(height ~ poly(age, 4), Oxboys, ~age +
+ I(age^2) | Subject), gc = TRUE)
[1] 0.11 0.00 0.11 0.00 0.00

> summary(mX2)

Linear mixed-effects model fit by REML
Fixed: height ~ poly(age, 4)
Data: Oxboys
      AIC      BIC    logLik
640.8686 682.3324 -308.4343

Random effects:

```

Groups	Name	Variance	Std.Dev.	Corr
Subject	(Intercept)	64.03464	8.00216	
	age	2.86418	1.69239	0.614
	I(age^2)	0.67429	0.82115	0.215 0.658
Residual		0.21737	0.46623	

Fixed effects:

	Estimate	Std. Error	DF	t value	Pr(> t )
(Intercept)	149.51976	1.59031	229	94.0194	< 2.2e-16
poly(age, 4)1	64.54095	3.32787	229	19.3941	< 2.2e-16
poly(age, 4)2	4.20322	1.02361	229	4.1063	5.597e-05
poly(age, 4)3	1.29077	0.46628	229	2.7682	0.006098
poly(age, 4)4	-0.58547	0.46630	229	-1.2556	0.210552

Correlation of Fixed Effects:

	(Intr)	p(,4)1	p(,4)2	p(,4)3
poly(ag,4)1	0.631			
poly(ag,4)2	0.230	0.583		
poly(ag,4)3	0.000	0.000	0.000	
poly(ag,4)4	0.000	0.000	0.000	0.000

Number of Observations: 234  
Number of Groups: 26

## 6 Cross-classification model

```
> data(ScotsSec)
> str(ScotsSec)
`data.frame':      3435 obs. of  6 variables:
 $ verbal : num  11 0 -14 -6 -30 -17 -17 -11 -9 -19 ...
 $ attain : num  10 3 2 3 2 2 4 6 4 2 ...
 $ primary: Factor w/ 148 levels "1","2","3","4",...: 1 1 1 1 1 1 1 1 1 1 ...
 $ sex    : Factor w/ 2 levels "M","F": 1 2 1 1 2 2 2 1 1 1 ...
 $ social : num  0 0 0 20 0 0 0 0 0 0 ...
 $ second : Factor w/ 19 levels "1","2","3","4",...: 9 9 9 9 9 9 1 1 9 9 ...
> system.time(mS1 <- lme(attain ~ sex, ScotsSec, ~1 | primary +
+   second))
[1] 0.07 0.00 0.07 0.00 0.00
> summary(mS1)
Linear mixed-effects model fit by REML
Fixed: attain ~ sex
Data: ScotsSec
      AIC      BIC    logLik
17137.91 17168.62 -8563.956

Random effects:
Groups   Name      Variance Std.Dev.
primary (Intercept) 1.10962  1.0534
second  (Intercept) 0.36966  0.6080
Residual                    8.0551  2.8382

Fixed effects:
      Estimate Std. Error   DF t value Pr(>|t|)
(Intercept) 5.2552e+00 1.8432e-01 3433 28.5108 < 2.2e-16
```

sexF 4.9851e-01 9.8255e-02 3433 5.0737 4.109e-07

Correlation of Fixed Effects:

(Intr)

sexF -0.264

Number of Observations: 3435

Number of Groups:

primary	second
148	19